数理解析研究所講究録
第 1805 巻 2012 年 25-36

Ground States for $2D$ Spin Glasses

Louis-Pierre Arguin
Université de Montréal, Canada

Michael Damron
Princeton University, USA

Charles M. Newman
New York University, USA

Daniel L. Stein
New York University, USA

Research supported in part by NSF grants DMS-0604869, OISE-0730136 and DMS-1106316 and an NSF postdoctoral fellowship to M. Damron.

Edwards-Anderson (EA) Spin Glass

For $G_N = (V_N, E_N)$, graph with N vertices, let

$$H_N(\sigma) = -\sum_{(x,y)\in E_N} J_{xy}\sigma_x\sigma_y\ .$$

with $J := \{J_{xy}\}$ indep. mean zero Gaussian (or other).

- For SK model [SK], G_N is the complete graph.
- For EA model [EA], $G_N \subset \mathbb{Z}^d$.
- Spin glasses exhibit many, very different, states of low energy that lead to complex behavior.
- How many ground states as $N \to \infty$?
- How different is EA than SK? For what d?

Gibbs States and Ground States

- The Gibbs measure is concentrated on σ's with small H_N.

$$\mathcal{G}_{\beta,N}(\sigma) = \frac{\exp -\beta H_N(\sigma)}{Z_N(\beta)}$$

- $\mathcal{G}_{\beta,N}$ gives information on σ's close to the minimum.

Take $G_N = (V_N, E_N)$, a box in $\mathbb{Z}^d$ (of volume N).

- Take periodic b.c. (so $H_N(\sigma) = H_N(-\sigma)$).
- Write $\alpha := \{\sigma, -\sigma\}$, a **Ground State Pair (GSP)**.
- Study the measure on the unique GSP $\alpha_N(J)$: $\delta_{\alpha_N(J)}$.
- Equivalent to studying Gibbs state with

$$\beta \to \infty, \text{ then } N \to \infty \; .$$

Metastate [AW, NS1]: a measure on GSP's

- $\alpha_{N,J}$ could change drastically between N and N'.
- Consider the joint distribution of $(\alpha_{N,J}, J)$,

$$K_N := \delta_{\alpha_{N,J}} \ \nu_N(dJ) \ .$$

- Take a (subsequence) limit of K_N as $N \to \infty$.
- Express the limit K as $K_J \ \nu(dJ)$.

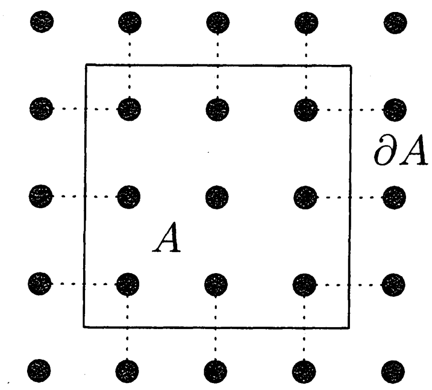
- $K_J \ \nu(dJ)$ is **translation invariant**. (Periodic b.c.)
- K_J is a measure on GSP's on $\mathbb{Z}^d$ for given J.

Ground States in Infinite Volume

Definition (Ground State Property)

$\alpha = \{\sigma, -\sigma\}$ is a GSP on $\mathbb{Z}^d$ for J if for any finite $A \subset \mathbb{Z}^d$

$$- \sum_{(x,y) \in \partial A} J_{xy} \sigma_x \sigma_y < 0 \ .$$



Ground States in Low Dimension

Conjecture (Uniqueness of Ground States)

For low d $(d < d_c = 6?, = 8?, = \infty?)$,

- *the limit* $K = K_J\,\nu(dJ)$ *exists (no subseq. needed);*
- K_J *is supported on a single GSP.*

($d = 2$ numerics of Palassini-Young [PY], Middleton [M])

Strategy:

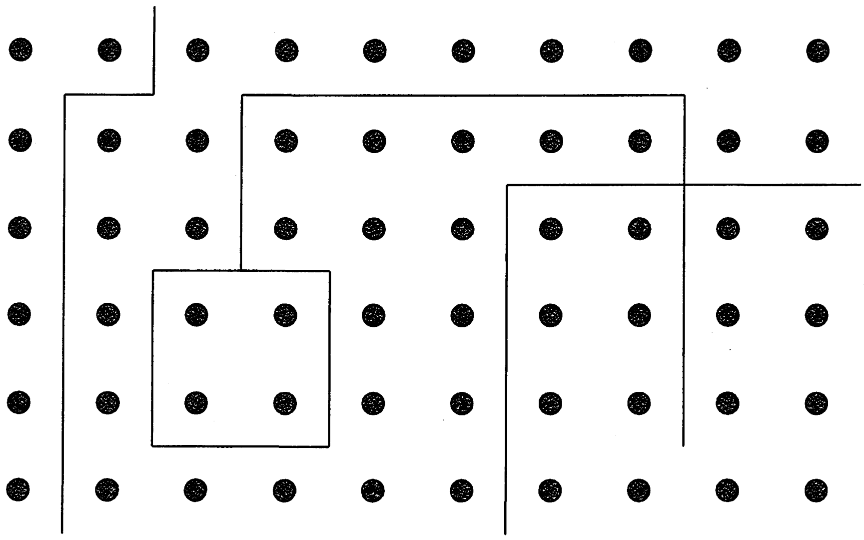
1. Let α and α' be replica GSP's.
2. Study the **interface**:

$$\alpha\Delta\alpha' := \{(x,y) : \alpha_{xy} \neq \alpha'_{xy}\}\ ,$$

   where $\alpha_{xy} := \sigma_x\sigma_y$.
3. Show $\alpha\Delta\alpha'$ is empty (hence $\alpha = \alpha'$).

Interfaces between GSP's

- $\alpha\Delta\alpha' = \{(x, y) : \alpha_{xy} \neq \alpha'_{xy}\}$.
- Put a dual edge whenever $(x, y) \in \alpha\Delta\alpha'$.



Full Plane Partial Result

If α and α' from metastate are distinct; then

- $\alpha \Delta \alpha'$ cannot have dangling ends (or 3-branching points).
- cannot contain loops.
- cannot have 4-branching points.

So $\alpha \Delta \alpha'$ is one or more doubly-infinite (self-avoiding) paths.

Theorem (Newman, Stein [NS2])

In fact, non-empty $\alpha \Delta \alpha'$ can only be a **single** *path.*

Uniqueness of GSP's in the Half-Plane

Take $G_N = [-N, N] \times [0, 2N] \cap \mathbb{Z}^2$ with horizontal periodic and vertical free b.c. and let $N \to \infty$.

Horizontal but not vertical translation invariance

Theorem (Arguin, Damron, Newman, Stein [ADNS])

In the half-plane,

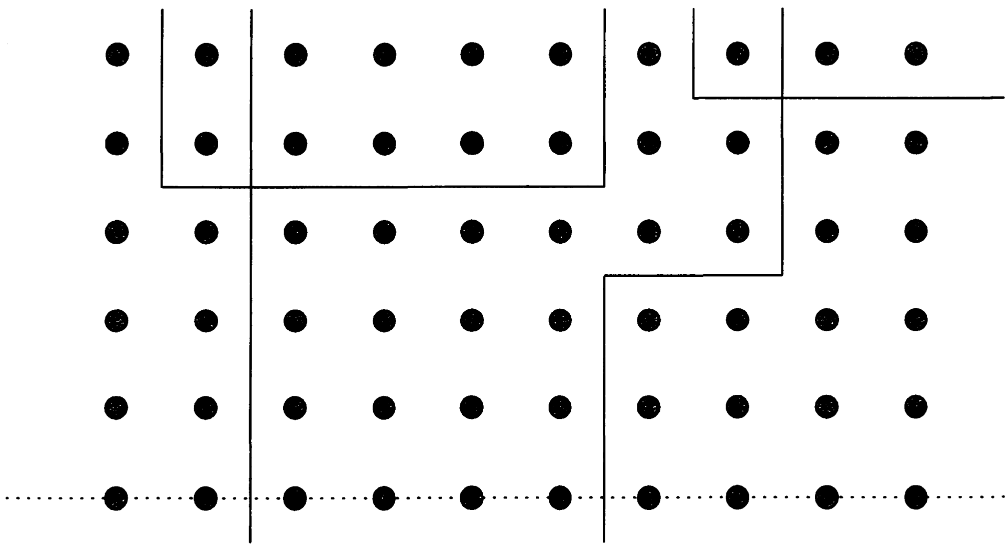
- *the limit* $K = K_J\ \nu(dJ)$ *exists (no subseq.);*
- K_J *is supported on a single GSP.*

This is the first complete result for $d > 1$.

Strategy — show interface between replicas must be empty.

Interface of GSP's in the Half-Plane

If $\alpha \Delta \alpha' \neq \emptyset$, then there are infinitely many **tethered paths**:



From Half to Full Plane

Let $\mu^* = \lim_{k\to\infty} \frac{1}{k} \sum_{l=1}^{k} T^{-l}\mu$ (along subseq.) where T is the vertical shift and μ is distrib. of (α, α', J).

- μ^* is a measure on the **full plane**,
- translation invariant in full plane by construction.

Because **we see many tethered paths**, if $\alpha\Delta\alpha' \neq \emptyset$, then $\alpha\Delta\alpha'$ is not a single path, contradicting full plane result of [NS2].

To do list for the future:

- Uniqueness of GSP's in the full plane.
- Uniqueness (or not) of GSP's for $d > 2$?
- Metastates applied to $\beta < \infty$.

References

[ADNS] Arguin, L.-P., Damron, M., Newman, C.M., Stein, D.L.: Commun. Math. Phys. **300**, 641–657 (2010)

[AW] Aizenman, M., Wehr, J.: Commun. Math. Phys. **130**, 489–528 (1990)

[EA] Edwards, S., Anderson, P.W.: J. Phys. F **5**, 965–974 (1975)

[M] Middleton, A.A.: Phys. Rev. Lett. **83**, 1672–1675 (1999)

[NS1] Newman, C.M., Stein, D.L.: Phys. Rev. Lett. **76**, 4821–4824 (1996)

[NS2] Newman, C.M., Stein, D.L.: Commun. Math. Phys. **224**, 205–218 (2001)

[PY] Palassini, M, Young, A.P.: Phys. Rev. B **60**, R9919–R9922 (1999)

[SK] Sherrington, D, Kirkpatrick, S.: Phys. Rev. Lett. **35**, 1792–1796 (1975)